

Ideals, Congruences and RDP on Unitizations of Generalized Pseudo Effect Algebras

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Abstract

A generalized pseudo effect algebra (GPEA) is a partially ordered partial algebraic structure that has the smallest element 0 and is not necessarily bounded from above. In [2] there has been shown that a GPEA P can be embedded as a maximal proper PEA-ideal in its unitization (which is a pseudo effect algebra - PEA - a structure with a unit) if and only if P admits a so-called unitizing GPEA-automorphism. This result made the former construction of a unitization of weakly commutative GPEA (used in [4]) just a special case (with the unitizing automorphism being the identity mapping).

In our former paper ([3]) we studied i.a. properties of Riesz ideals and congruences according to their inheritance from a generalized effect algebra to its unitization effect algebra. In this contribution (based on [1]) we generalize these results for pseudo (non-commutative) structures of GPEAs and their unitizations through the unitizing automorphisms. In particular, we state conditions under which a congruence of a GPEA P can be extended to a congruence on its γ -unitization U (where γ is the unitizing automorphism on P) such that the quotient of U is the unitization of the quotient of P with a unitizing automorphism induced by γ . We will also show how the Riesz decomposition properties are preserved in this setting.

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References

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