Contact algebra as a tool for modal logic

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A contact algebra $\underline{B} = (B, \mathcal{C})$ is a Boolean algebra together with a binary relation $\mathcal{C}$ (called contact relation) which satisfies $(a, b \in B)$

1. for each $a \neq 0$, the sets $\mathcal{C}(a, -) = \{b \mid a \; \mathcal{C} \; b\}$ et $\mathcal{C}(-, a) = \{b \mid b \; \mathcal{C} \; a\}$ are proper filters,
2. if $a \neq 0$, then $a \; \mathcal{C} \; a$
3. $\mathcal{C}$ is symmetric.

Contact algebras have been introduced as a model for qualitative spatial reasoning and may be realized as Boolean algebras of regular closed subsets of topological spaces, where being in contact meaning having non empty intersection.

Rather surprisingly but almost obviously, the canonical extension $\underline{B}^\delta$ of a contact algebra $\underline{B}$ may naturally be considered as a modal algebra, so that, for a modal (propositional) formula φ, the validity

$$\underline{B} \models \varphi$$

may be defined by stipulating that if v is a valuation on B (that is v is a mapping from the set Var of variables into B), the unique homomorphic extension v^δ of v to $\underline{B}^\delta$ satisfies $v^\delta(\varphi) = 1$.

Various concepts, problems and results of modal logic may be transferred (in fact extended) to the contact setting, and it is the aim of this talk to examine some of them :

1. which first order properties in the contact language are modally definable ?
2. for which modal logic do we have completeness results with respect to the contact algebra semantic ?
3. is there something like contact canonicity ?
4. what are the contact varieties, that is modal equationnally definable classes of contact algebras ?