On injective constructions of S-semigroups

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There are quite a lot of papers investigating injective hulls for algebras. Here we only mention some of them which our research is related to. Injective hulls for posets were studied by Banaschewski and Bruns ([1], 1967) where they got that the injective hull of a poset is its MacNeille completion. Later on, Bruns and Lakser constructed injective hulls of semilattices ([2], 1970), and the results were soon applied into S-systems over a semilattice by Johnson, Jr., and McMorris ([3], 1972). In 2012, Lambek, Barr, Kennison, Raphael ([4]) studied a kind of category of pomonoids in which normal category of pomonoids is its subcategory, and found injective hulls for pomonoids. Later on, Zhang and Laan generalized the results of Lambek, Barr, Kennison, Raphael into posemigroup case ([5], 2014), and then S-posets ([6], 2015), ordered Ω-algebras ([7], 2016).

This paper devotes to investigations on injective constructions of S-semigroups. We will study injectivity with respect to a specific class of order-embeddings.

Our approach in this paper sheds a new light on applications of a new kind of quantale-like structure and establishes the injective hull for every S-semigroup.

References

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