

Meet-irreducible congruence lattices of algebras with small cycles

Lucia Janičková*

P.J. Šafárik University in Košice, Slovakia

*joint work with D. Jakubíková–Studenovská (P.J. Šafárik University)

Abstract

It is known that all congruences of an algebra \mathcal{A} ordered by inclusion form a lattice, denoted $\text{Con } \mathcal{A}$. In 1963, it was proved by G. Grätzer and E.T. Schmidt that this lattice is algebraic and that every algebraic lattice is isomorphic to the congruence lattice of some algebra.

Further, for a given set A , the system of all $\text{Con } \mathcal{A}$, where \mathcal{A} is an algebra with the base set A , forms a lattice (with respect to class-theoretical inclusion).

Let \mathcal{E}_A be the lattice of all congruence lattices $\text{Con } \mathcal{A}$. It is known that this lattice is atomistic and if $|A| \geq 4$, it is tolerance simple. Also, all join-irreducible congruence lattices were characterized. The question which elements of \mathcal{E}_A are meet-irreducible is only partially answered.

Since $F \subseteq G$ implies $\text{Con}(A, G) \subseteq \text{Con}(A, F)$, all meet-irreducible elements in \mathcal{E}_A must be of the form $\text{Con}(A, f)$ for a single mapping f , otherwise $\text{Con}(A, F)$ would be the intersection of all $\text{Con}(A, f)$ where $f \in F$. Therefore, it is sufficient to explore meet-irreducibility of congruence lattices of monounary algebras.

In 2017, Studenovská, Pöschel and Radeleczki presented some partial answers to the question which lattices $\text{Con}(A, f)$ for a given finite set A are meet-irreducible, namely, in the case when each cycle contains only one element and in the case when f is a permutation. Also, the coatoms of \mathcal{E}_A are known.

Our aim is to contribute to the characterization of meet-irreducible elements of \mathcal{E}_A . We study the meet-irreducible elements of \mathcal{E}_A in the case that every cycle of the algebra \mathcal{A} has at most two elements, i.e. \mathcal{A} is an algebra with small cycles.

We proved the necessary and sufficient conditions under which $\text{Con}(A, f)$ is meet-irreducible in the case that (A, f) is an algebra with small cycles.