Lattice valued structures

Andreja Tepavčević

University of Novi Sad, Faculty of Sciences, Department of Mathematics and Informatics, Novi Sad, Serbia

Mathematical Institute of the Serbian Academy of Sciences and Arts, Belgrade, Serbia

The aim of the talk is to present the development and the present state-of-the-art in the field of lattice valued relational and algebraic structures.

The topic originates in works of Scott and Solovay (late sixties of the 20th century) when the Boolean-valued models were constructed, with the initial intention to prove the independence results of the set theory. Later on, the omega sets were introduced by Fourman and Scott. The construction was similar, still a Boolean lattice was replaced by a Heyting algebra. Here, the reason was to model intuitionistic logic. In both cases an algebraization of logic was performed by a particular ordered structure. Almost independently, the fuzzy mathematics was created, developing basic tools, closely related to the unit interval and real functions. Soon a fuzzy logic appeared, firstly on the intuitive level, then gradually covering all aspects of fuzziness, in particular when (by Goguen) the unit interval as the membership values structure was extended to a complete lattice. In addition, soon in this period (nineties of the 20th century), residuated lattices were accepted and used for the algebraization of the fuzzy logic. Different types of these lattices correspond to particular logic and set theoretic operators in fuzzines. Then also MV algebras have been (and still are) investigated in the same context. Connection to the Scott and Fourman approach was recognized and analyzed. Finally the classical equality was replaced by a generalized, fuzzy one (Höhle, Bělohlávek, Demirci and others).

Fuzzy algebraic structures were investigated from the beginning of the fuzzy era, mostly in the framework of generalized substructures, keeping the membership values structure to be the unit interval, then a complete lattice. Practically all known algebraic structures were analyzed from this aspect, by generalizing substructures to suitably compatible functions (semigroups, groups, Boolean and related algebras, ordered structures etc.). In this approach, axioms were translated to lattice theoretic formulas. An important property of these functions are cut sets, which are a link to classical structures, being classical semigroups, groups etc.

Introduction of a fuzzy equality instead of the classical one, has led to a new approach to graded structures. Algebras with fuzzy equalities became a topic of interest, and were extensively investigated in the recent decades (firstly by Bělohlávek, Vychodil, Demirci and then by many authors). All basic aspects of the universal algebra were appropriately covered. The corresponding logic was mostly supported by suitable residuated lattices.

Our approach to lattice valued structures keeps generalized equality, but, following Fourman and Scott, our reflexivity is weak, hence connecting cuts of the generalized equivalence with congruences on classical subalgebras. We remain at the cutworthy approach, therefore dealing with a complete lattice or a Heyting algebra as the membership values structure.

Main results in the present theory of lattice valued algebras with generalized equalities and some applications will be elaborated.