Canonical extensions of lattices are more than perfect

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Canonical extensions of lattice-based algebras originated in the 1951-52 papers of B. Jónsson and A. Tarski on Boolean algebras with operators. When the members of a variety of lattice-based algebras are algebraic models of a logic, canonicity (meaning that algebraic identities are preserved when constructing canonical extensions of algebras) leads to completeness for the associated logic. The concept has been intensively studied for distributive lattice expansions, and more generally for lattice and even poset expansions, during the last 25 years (M. Gehrke, J. Harding, B. Jónsson, A. Palmigiano, H.A. Priestley, Y. Venema, and others). We present a brief overview of the studies of the canonical extensions of lattices.

We then present results of our work where we consider properties of the graphs that arise as duals of bounded lattices in Ploščica's representation (1995) via maximal partial maps into the two-element set (this recasts Urquhart's representation from 1978 in the spirit of the natural dualities). We introduce TiRS graphs which abstract the considered lattice duals. We demonstrate a one-to-one correspondence of TiRS graphs with so-called TiRS frames which are a subclass of the class of RS frames introduced by Gehrke (2006) to represent perfect lattices. This yields a dual representation of finite lattices via finite TiRS frames, or equivalently finite TiRS graphs, which generalises the well-known Birkhoff dual representation of finite distributive lattices via finite posets from the 1930s.

We indicate our recent investigations in this topic. We show that the canonical extensions of lattices are more than perfect - they are perfect lattices satisfying an extra (PTi) condition. Thus they belong to a newly introduced class of (PTi) lattices that correspond to our TiRS frames (resp. TiRS graphs). We also show that there are (PTi) lattices that are not the canonical extension of any bounded lattice. On the side of the relational structures this extends the notion of a non-representable poset to the duals of non-distributive lattices. We present questions that can be of interest to researchers in this and related areas.