ON MODULAR AND CANCELLABLE ELEMENTS OF THE LATTICE OF EPIGROUP VARIETIES

DMITRY SKOKOV

An *epigroup* is a semigroup in which some power of any element lies in a subgroup of the given semigroup. Epigroups can be treated as unary semigroups, that is semigroups equipped by an additional unary operation defined by the following way. If S is an epigroup and $x \in S$ then some power of x lies in a maximal subgroup of S. We denote this subgroup by G_x and the unit element of G_x by x^ω. It is well known that the element x^ω is well defined and $xx^\omega = x^\omega x \in G_x$. We denote the element inverse to xx^ω in G_x by $\overline{x}$. The map $x \longrightarrow \overline{x}$ is just the mentioned unary operation on an epigroup S. The element $\overline{x}$ is called *pseudoinverse* to x. So, we can consider varieties of epigroups as algebras with two operations, namely multiplication and pseudoinversion. An idea to examine epigroups in the framework of the theory of varieties was promoted by L.N. Shevrin in [2, 3].

An element x of a lattice $\langle L; \vee, \wedge\rangle$ is called

modular if $\quad (\forall y, z \in L) \quad \big(y \le z \longrightarrow (x \vee y) \wedge z = (x \wedge z) \vee y\big)$,

cancellable if $\quad (\forall y, z \in L) \quad (x \vee y = x \vee z \;\&\; x \wedge y = x \wedge z \longrightarrow y = z)$.

It is easy to see that a cancellable element is a modular one. Modular elements of the lattice $\mathbb{EPI}$ of all epigroup varieties were examined in [1]. In particular, commutative epigroup varieties that are modular elements of $\mathbb{EPI}$ were completely determined there. Cancellable elements of $\mathbb{EPI}$ were not examined so far.

In this work we completely determine all commutative epigroup varieties that are cancellable elements of $\mathbb{EPI}$. In particular, we verify that a commutative epigroup variety is a cancellable element of the lattice $\mathbb{EPI}$ if and only if it is a modular element of this lattice. This equivalence is false in slightly wider class, namely in the class of all varieties that satisfies a permutational identity of length 3, that is an identity of the form $x_1x_2x_3 = x_{1\pi}x_{2\pi}x_{3\pi}$ where π is a non-trivial permutation on the set $\{1, 2, 3\}$.

The work is partially supported by Russian Foundation for Basic Research (grant 17-01-00551) and by the Ministry of Education and Science of the Russian Federation (project 1.6018.2017/8.9).

References

[1] V.Yu.Shaprynskiĭ , D.V.Skokov, B.M.Vernikov. *Special elements of the lattice of epigroup varieties*, Algebra Universalis. **75** (3), 1–30 (2016).

[2] L. N. Shevrin. *On theory of epigroups. I, II* // Russ. Math. Sb. 1995. Vol. 82. P. 485–512. 1995. Vol. 83. P. 133–154.

[3] L. N. Shevrin. *Epigroups*, in V. B. Kudryavtsev and I. G. Rosenberg (eds.), Structural Theory of Automata, Semigroups, and Universal Algebra, Dordrecht, Springer. 2005. P. 331–380.

Ural Federal University, Ekaterinburg
E-mail address: dmitry.skokov@gmail.com