Some Extremal Values of the Number of Congruences of a Finite Lattice

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We study the numbers of congruences of finite lattices. Regarding the smallest values of these numbers, we prove that, for any natural number $n \geq 7$, any $j \in \overline{1, n-1}$ and any $k \in \overline{2, n+1}$, there exists an n–element lattice with exactly 2^j congruences and, if $n \neq 8$, then there exists an n–element lattice with exactly k congruences. Regarding the largest values of these numbers, it turns out that, for any nonzero natural number n, the five largest numbers of congruences of an n–element lattice are, in descending order, 2^{n-1}, 2^{n-2}, $5 \cdot 2^{n-5}$, 2^{n-3} and $7 \cdot 2^{n-6}$, provided there exist n–element lattices with exactly these values for the cardinalities of their sets of congruences. The first of these values is only reached by the n–element chain, according to [1, 2], and the second is only reached by the ordinal sum of a finite chain with the four–element Boolean algebra and another finite chain, whenever $n \geq 4$, by [1]. We prove that the next largest three values are as stated above and we also show the shapes of the n–element lattices having these numbers of congruences, which exist for any $n \geq 5$ in the case of the third largest number and for any $n \geq 6$ in the cases of the fourth and fifth.

References

[1] G. Czédli, A Note on Finite Lattices with Many Congruences, arXiv:1712.06117 [math.RA].

[2] R. Freese, Computing Congruence Lattices of Finite Lattices, *Proc. Amer. Math. Soc.* **125**, 3457–3463 (1997).