

# On the lattice of closed subspaces of a complex Hilbert space

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The basic model of quantum physics is the Hilbert space over the complex numbers. Its algebraic characterisation could be of interest to bring light to the old question why precisely this model has been so successful.

Due to a result of W. J. Wilbur from the 1970s, the lattice of closed subspaces of the infinite-dimensional complex Hilbert space can be characterised (practically) purely lattice-theoretically [3]. However, several conditions are technically involved and hard to interpret. The situation becomes more transparent if we refrain from the restriction to equations that hold in this lattice but take its automorphism group into account as well. We shall see that then a much more compact formulation is possible.

Furthermore, the set of closed subspaces of a Hilbert space may be characterised on the basis of alternative structures. Restricting the lattice-theoretic operations to compatible subspaces, we arrive at a partial Boolean algebra [1]. Furthermore, the set of unit vectors of the Hilbert space together with the orthogonality relation is an orthogonality space. In both cases, conditions on the existence of automorphisms support the characterisation of the structure under consideration quite efficiently.

This is joint work with Jan Paseka.

## References

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