MINORS OF MULTISORTED FUNCTIONS AND REFLECTIONS

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Let S be a set of elements called *sorts,* and let $A = (A_s)_{s \in S}$ be a family of sets. An *S-sorted operation* on A is a map $f\colon A_w \to A_s$, where $s \in S$, $w = w_1 w_2 \dots w_n$ is a word over the alphabet S, and $A_w = A_{w_1} \times A_{w_2} \times \dots \times A_{w_n}$. The special case when S is a singleton corresponds to the usual (one-sorted) operations on a set A.

Let $f\colon A_w \to A_s$ and $g\colon A_u \to A_s$ be S-sorted operations on A with $w = w_1 w_2 \dots w_n$, $u = u_1 u_2 \dots u_m$. We say that f is a *minor* of g, and we write $f \leq g$, if $\{u_1, u_2, \dots, u_m\} \subseteq \{w_1, w_2, \dots, w_n\}$ and there exists a map $\lambda\colon \{1, 2, \dots, m\} \to \{1, 2, \dots, n\}$ such that $u_i = w_{\lambda(i)}$ for all $i \in \{1, 2, \dots, m\}$ and $f(a_1, a_2, \dots, a_n) = g(a_{\lambda(1)}, a_{\lambda(2)}, \dots, a_{\lambda(m)})$ for all $(a_1, a_2, \dots, a_n) \in A_w$. The minor relation $\leq$ is a quasi-order on the set $\mathcal{F}_A$ of all S-sorted operations on A, and it induces an equivalence relation $\equiv$ on $\mathcal{F}_A$ and a partial order $\leq$ on $\mathcal{F}_A/{\equiv}$ in the usual way.

We investigate minors of multisorted operations from different points of view. On the one hand, we provide structural descriptions of fragments of the minor poset. On the other hand, we generalize Pippenger's [2] Galois theory of minor-closed classes and relation pairs to the multisorted setting. We also show how reflections (as defined by Barto, Opršal, Pinsker [1]) interact with minors.

This is joint work with Reinhard Pöschel and Tamás Waldhauser.

References

[1] L. Barto, J. Opršal, M. Pinsker, The wonderland of reflections, *Isr. J. Math.* (2017), DOI: 10.1007/s11856-017-1621-9.

[2] N. Pippenger, Galois theory for minors of finite functions, *Discrete Math.* **254** (2002) 405–419.