Classification of semigroups of order three by "identities" introduced by Lyapin

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Abstract

Algebraic structures can be described by varieties in the sense of Birkhoff. It is well known that neither the class of all semigroups of order three nor any of its subclasses forms a variety. Therefore, we will use a concept, introduced by Lyapin [2] and Evseev [1], respectively, in order to describe this class. Let $X := \{x_1, x_2, \ldots\}$ be a countable set of variables and let X^+ be the set of all words over the set X. An expression $u \approx v$ (with $u, v \in X^+$) will be called equation and let $Eq(X)$ be the set of all possible equations. A subset $\sigma \subseteq Eq(X)$ is called disjunction of identities [3]. We say that σ is satisfied by a semigroup S (in symbols: $S \models \sigma$) if for all mappings $h : X \to S$, there is an equation $u \approx v \in \sigma$ such that $\overline{h}(u) = \overline{h}(v)$, where $\overline{h} : X^+ \to S$ denotes the unique determined homomorphic extension of h to the word semigroup X^+. Let Σ be a set of disjunction of identities, i.e. Σ is a subset of the power set of $Eq(X)$. Then we put $Mod\Sigma$ as the class of all semigroups S with $S \models \sigma$ for all $\sigma \in \Sigma$. We will call such a class $Mod\Sigma$ an alternative variety. An alternative variety is an generalization of the classical concept of a variety due to Birkhoff, namely taking Σ being a set of singleton sets. In particular, $Mod\Sigma$ is closed under homomorphic images.

We will classify all non-group semigroups of three by exactly one disjunction of identities.

References

[1] Evseev, A.E., "Semigroups with Some Power Identity Inclusions, Algebraic Systems with One Operation and One Relation", Interuniv Collect. sci. Works, Leningrad, 1985, 21-32.

[2] Lyapin, E.S., "Identities Valid Globally in Semigroups", Semigroup Forum 24 Issue 1 (1982), 263-269.

[3] Thron, R., and Koppitz, J.„ "Finite Relational Disjunctions", Algebra Colloquium 8 (3) (1998), 261-268.