EMV-ALGEBRAS, STATE-MORPHISMS AND THE LOOMIS–SIKORSKI THEOREM

ANATOLIJ DVUREČENSKIJ[1,2]

[1] Mathematical Institute, Slovak Academy of Sciences
Štefánikova 49, SK-814 73 Bratislava, Slovakia
[2] Depart. Algebra Geom., Palacký Univer.
17. listopadu 12, CZ-771 46 Olomouc, Czech Republic
E-mail: dvurecen@mat.savba.sk

Recently in [DvZa1], new algebraic structures, EMV-algebras, were introduced. They generalize MV-algebras in such a sense as generalized Boolean do Boolean algebras. Such algebras behave locally as MV-algebras, i.e. for each idempotent $a \in M$, the interval $[0, a]$ forms an MV-algebra and in general, a top element of an EMV-algebra is not assumed.

Let $(M; \oplus, 0)$ be a commutative monoid. An element $a \in M$ is said to be *idempotent* if $a \oplus a = a$. We denote by $\mathcal{I}(M)$ the set of idempotents of M.

An algebra $(M; \vee, \wedge, \oplus, 0)$ of type $(2, 2, 2, 0)$ is called an *extended MV-algebra*, an *EMV-algebra* in short, if it satisfies the following conditions:

(EM1) $(M; \vee, \wedge, 0)$ is a distributive lattice with the least element 0;
(EM2) $(M; \oplus, 0)$ is a commutative ordered monoid with neutral element 0;
(EM3) for each $b \in \mathcal{I}(M)$, the element

$$\lambda_b(x) = \min\{z \in [0, b] \mid x \oplus z = b\}$$

exists in M for all $x \in [0, b]$, and the algebra $([0, b]; \oplus, \lambda_b, 0, b)$ is an MV-algebra;
(EM4) for each $x \in M$, there is $a \in \mathcal{I}(M)$ such that $x \leq a$.

An EMV-algebra M is called *proper* if M has not a top element. If $(M; \oplus, ', 0, 1)$ is an MV-algebra, then $(M; \vee, \wedge, \oplus, 0)$ is an EMV-algebra with top element 1, and conversely, if $(M; \vee, \wedge, \oplus, 0)$ is an EMV-algebra with top element 1, then $(M; \oplus, ', 0, 1)$ is an MV-algebra, so that EMV-algebras with top element are termwise equivalent to MV-algebras.

The basic representation theorem of EMV-algebras, [DvZa1, Thm 5.21], says that every EMV-algebra M is either an EMV-algebra with top element or it is a maximal ideal of an EMV-algebra N with top element, and every element of N either belongs to M or it is a complement of some element of M. It generalizes analogous result for generalized Boolean algebras from [CoDa, Thm. 2.2].

We present basic properties of EMV-algebras, examples like EMV-clans and EMV-tribes consisting of fuzzy sets with pointwise defined EMV-operations. We note that a *state-morphism* is an EMV-homomorphism s from M into the EMV-algebra of the real interval $[0, 1]$ such that $s(x) = 1$ for some element $x \in M$. State-morphisms exist in each non-trivial EMV-algebra and they form a locally compact Hausdorf topological space whose one-point compactification corresponds to the state-morphism space of the representing EMV-algebra with top element.

[1]The paper has been supported by the grant of the Slovak Research and Development Agency under contract APVV-16-0073, by the grants VEGA No. 2/0069/16 SAV, and GAČR 15-15286S.

State-morphisms enable to prove the Loomis–Sikorski theorem for σ-complete EMV-algebras, see [DvZa2]:

Theorem 0.1. *Let M be a σ-complete EMV-algebra. Then there are an EMV-tribe $\mathcal{T}$ of fuzzy sets on some $\Omega \neq \emptyset$ and a surjective σ-homomorphism h of EMV-algebras from $\mathcal{T}$ onto M.*

This is a joint work with Omid Zahiri.

References

[CoDa] P. Conrad, M.R. Darnel, Generalized Boolean algebras in lattice-ordered groups, *Order*, **14** (1998), 295–319.

[DvZa1] A. Dvurečenskij, O. Zahiri, *On EMV-algebras*, Inter. J. Approx. Reasoning

[DvZa2] A. Dvurečenskij, O. Zahiri, *The Loomis–Sikorski theorem for EMV-algebras*, J. Austral. Math. Soc.