Weakly orthomodular and dually weakly orthomodular lattices

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Abstract

We introduce so-called weakly orthomodular and dually weakly orthomodular lattices which are lattices with a unary operation satisfying formally the orthomodular law or its dual although neither boundedness nor complementation is assumed. It turns out that lattices being both weakly orthomodular and dually weakly orthomodular are in fact complemented but the complementation need not be neither antitone nor an involution. Moreover, every modular lattice with complementation is both weakly orthomodular and dually weakly orthomodular. The class of weakly orthomodular lattices and the class of dually weakly orthomodular lattices form varieties which are arithmetical and congruence regular. Connections to left residuated lattices are presented and commuting elements are introduced. Using commuting elements, we define a center of such a (dually) weakly orthomodular lattice and we provide conditions under which such lattices can be represented as a non-trivial direct product.

Keywords: weakly orthomodular lattice, orthomodular lattice, lattice with complementation, residuated lattice, congruence permutability, congruence regularity

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