$\mathcal{I}$-CONTINUITY IN TOPOLOGICAL SPACES

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Abstract. In this paper we generalize the notion of $\mathcal{I}$-continuity, which was defined in [1] for real functions, to maps on topological spaces. We study the classes of topological spaces such that for each map on X $\mathcal{I}$-continuity implies continuity.

Introduction

This paper was inspired by [1], where the notion of $\mathcal{I}$-continuity is defined for real functions. We generalize this notion to functions on arbitrary topological spaces.

It was shown in [7, Proposition 3.3] that for metric spaces $\mathcal{I}$-continuity and continuity are equivalent. We show that they are equivalent for sequential spaces as well.

The aim of this paper is to characterize the class of all topological spaces X such that $\mathcal{I}$-convergence and convergence are equivalent for functions on X. When studying this class, we associate with each proper ideal $\mathcal{I}$ in $\mathbb{N}$ a prime space $\mathbb{N}_{\mathcal{I}}$ and show how the $\mathcal{I}$-convergence of sequences in X is related to the continuity of maps from $\mathbb{N}_{\mathcal{I}}$ to X.

1. Definitions

The notion of $\mathcal{I}$-convergence was defined in [7] for sequences in metric spaces. It was used in [1] to introduce $\mathcal{I}$-convergence for real functions. These notions can be simply generalized for functions and sequences in topological spaces.

Definition 1.1. A family $\mathcal{I}$ of subsets of $\mathbb{N}$ is an *ideal* in $\mathbb{N}$ if

(1) $A, B \in \mathcal{I} \Rightarrow A \cup B \in \mathcal{I}$,
(2) $A \in \mathcal{I}$ and $B \subset A \Rightarrow B \in \mathcal{I}$.

Let us call an ideal $\mathcal{I}$ in $\mathbb{N}$ *proper* if $\mathbb{N} \notin \mathcal{I}$. $\mathcal{I}$ is *admissible* if $\mathcal{I}$ is proper and it contains every singleton. If $\mathcal{I}$ is an ideal in $\mathbb{N}$ then $\mathcal{F}(\mathcal{I}) = \{A : \mathbb{N} \setminus A \in \mathcal{I}\}$ is the *filter associated with the ideal $\mathcal{I}$*.

Definition 1.2. Let $\mathcal{I}$ be an ideal in $\mathbb{N}$. A sequence $(x_n)_{n=1}^{\infty}$ in a topological space X is said to be *$\mathcal{I}$-convergent* to a point $x \in X$ if

$$A(U) = \{n : x_n \notin U\} \in \mathcal{I}$$

holds for each open neighborhood U of x. We denote it by $\mathcal{I}\text{-}\lim x_n = x$.

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If $\mathcal{I} = \mathcal{I}_f$ is the Fréchet ideal (i.e. the ideal containing exactly all finite subsets of $\mathbb{N}$) then $\mathcal{I}$-convergence coincides with the usual convergence.

If $\mathcal{I}$ is an admissible ideal then $\mathcal{I}_f \subset \mathcal{I}$. So $\mathcal{I}_f$-$\lim x_n = x$ implies $\mathcal{I}$-$\lim x_n = x$.

One can see easily that if X is a Hausdorff space and $\mathcal{I}$ is a proper ideal in $\mathbb{N}$ then $\mathcal{I}$-$\lim x_n$ (if exists) is determined uniquely. (This does not hold in general.)

Using the concept of $\mathcal{I}$-convergence we can define $\mathcal{I}$-continuity in the way analogous to the Heine definition of limit of a function at a point.

Definition 1.3. Let $\mathcal{I}$ be an ideal in $\mathbb{N}$ and X, Y be topological spaces. A map $f\colon X \to Y$ is called *$\mathcal{I}$-continuous* if for each sequence $(x_n)_{n=1}^{\infty}$ in X

$$\mathcal{I}\text{-}\lim x_n = x_0 \quad \Rightarrow \quad \mathcal{I}\text{-}\lim f(x_n) = f(x_0)$$

holds.

We recall some basic facts concerning coreflective subcategories of the category **Top** of topological spaces. *Coreflective subcategories* of **Top** are classes of topological spaces which are closed under the formation of topological sums and quotient spaces. We will deal with some such classes in this paper. Therefore some results can be formulated more simply using coreflective subcategories of **Top**. No prior knowledge of them is required.

If **A** is a class of topological spaces, then the *coreflective hull* CH(**A**) of **A** is the smallest coreflective subcategory of **Top** which contains **A**. A topological space X belongs to CH(**A**) if and only if it is a quotient space of a topological sum of spaces from **A**.

A topological space X is *finitely generated* if every intersection of open subsets of X is open. The class **FG** is the coreflective hull of the Sierpiński space S. S is the space on the set $\{0, 1\}$ with open sets $\emptyset$, $\{0\}$ and $\{0, 1\}$.

2. Basic properties of $\mathcal{I}$-continuity

In this section we formulate some basic results concerning $\mathcal{I}$-continuity. We show that for sequential spaces continuity, $\mathcal{I}$-continuity and $\mathcal{I}_f$-continuity are equivalent.

It is easy to see that the following proposition holds:

Proposition 2.1. *If $f\colon X \to Y$ and $g\colon Y \to Z$ are $\mathcal{I}$-continuous, then $g \circ f$ is also $\mathcal{I}$-continuous.*

The following two proofs are slightly modified proofs of [7, Proposition 3.3] and [1, Theorem 1].

Theorem 2.2. *Let X, Y be topological spaces and let $\mathcal{I}$ be an arbitrary ideal in $\mathbb{N}$. If $f\colon X \to Y$ is continuous then f is $\mathcal{I}$-continuous.*

Proof. Let $f\colon X \to Y$ be continuous and $\mathcal{I}$-$\lim x_n = x$. Then for each open neighborhood V of $f(x)$ there exists an open neighborhood U of x such that $f[U] \subset V$. Hence

$$\{n \in \mathbb{N} : f(x_n) \notin V\} \subset \{n \in \mathbb{N} : x_n \notin U\} \in \mathcal{I}$$

and $\mathcal{I}$-$\lim f(x_n) = f(x)$.

Theorem 2.3. *Let X, Y be topological spaces and let $\mathcal{I}$ be an arbitrary admissible ideal. If $f\colon X \to Y$ is $\mathcal{I}$-continuous then f is $\mathcal{I}_f$-continuous.*

Proof. Assume that f is $\mathcal{I}$-continuous but it is not $\mathcal{I}_f$-continuous. Then there exists a sequence $(x_n)_{n=1}^{\infty}$ such that $\mathcal{I}_f$-$\lim x_n = x$ but it does not hold $\mathcal{I}_f$-$\lim f(x_n) = f(x)$. So there exists an open neighborhood V of $f(x)$ such that $A(V) = \{n : f(x_n) \notin V\} \notin \mathcal{I}_f$ i.e. $A(V)$ is infinite. Let $(y_n)_{n=1}^{\infty}$ be the subsequence of $(x_n)_{n=1}^{\infty}$ given by the subset $A(V)$ of $\mathbb{N}$. Then $\{n : f(y_n) \notin V\} = \mathbb{N}$. Also for the subsequence $(y_n)_{n=1}^{\infty}$ holds $\mathcal{I}_f$-$\lim y_n = x$. Then $\mathcal{I}$-$\lim y_n = x$ and, by $\mathcal{I}$-continuity of f, $\mathcal{I}$-$\lim f(y_n) = f(x)$. Hence $\{n : f(y_n) \notin V\} = \mathbb{N} \in \mathcal{I}$, a contradiction.

An example which shows that implications in Theorem 2.2 and Theorem 2.3 are not reversible will be given in the next section.

Topological spaces for which $\mathcal{I}_f$-continuity is equivalent to continuity are called *sequential spaces.* Sequential spaces were first thoroughly examined by S. P. Franklin in [4]. We recall some well-known facts about sequential spaces (see also [3], [6]). All first countable and all metric spaces are sequential. It is known that X is sequential if and only if a set $V \subset X$ is closed in X whenever it contains with each convergent sequence all its limits. Sequential spaces are exactly quotient spaces of metric spaces. Sequential spaces are closed under the formation of topological sums and quotient spaces, i.e. they form a coreflective subcategory of **Top**.

Corollary 2.4. *Let X be a sequential space and let $\mathcal{I}$ be an admissible ideal. Let Y be a topological space and let $f\colon X \to Y$ be a map. Then the following statements are equivalent:*

(1) *f is continuous,*
(2) *f is $\mathcal{I}_f$-continuous,*
(3) *f is $\mathcal{I}$-continuous.*

Special case of Corollary 2.4 is [7, Proposition 3.3], where it is proved for metric spaces.

3. $\mathcal{I}$-continuity and prime spaces

We say that a topological space X is a *prime space* if X has only one accumulation point (see [5], [2]). There exists an one-to-one correspondence between the prime spaces on the set $\mathbb{N} \cup \{\infty\}$ with the accumulation point ∞ and proper ideals in $\mathbb{N}$.

Let $\mathcal{I}$ be a proper ideal. We define a topological space $\mathbb{N}_{\mathcal{I}}$ on the set $\mathbb{N} \cup \{\infty\}$ as follows: $U \subset \mathbb{N} \cup \{\infty\}$ is open in $\mathbb{N}_{\mathcal{I}}$ if and only if $\infty \notin U$ or $U \setminus \{\infty\} \in \mathcal{F}(\mathcal{I})$. $\mathbb{N}_{\mathcal{I}}$ is clearly a prime space with the accumulation point ∞.

On the other hand, let P be a prime space on $\mathbb{N} \cup \{\infty\}$ with the accumulation point ∞. Let $\mathcal{I} = \{U \subset \mathbb{N} : U \text{ is closed in } P\}$. It is easy to see that $\mathcal{I}$ is a proper ideal.

Admissible ideals correspond to prime spaces in which every one-point set is closed, i.e. T_1-prime spaces. Any T_1-prime space is also Hausdorff.

The following result shows the relation between $\mathcal{I}$-convergence and the space $\mathbb{N}_{\mathcal{I}}$.

Proposition 3.1. *Let X be a topological space, $x \in X$, $x_n \in X$ for each $n \in \mathbb{N}$. Let us define a map $f\colon \mathbb{N}_{\mathcal{I}} \to X$ by $f(n) = x_n$ and $f(\infty) = x$. Then $\mathcal{I}$-$\lim x_n = x$ if and only if f is continuous.*

Proof. Let $\mathcal{I}$-$\lim x_n = x$ and let U be an open subset of X. If $x \notin U$ then $\infty \notin f^{-1}[U]$ and $f^{-1}[U]$ is open. If $x \in U$ then $\{n : f(n) \in U\} \in \mathcal{F}(\mathcal{I})$ and $f^{-1}[U] = \{n : f(n) \in U\} \cup \{\infty\}$ is open. So f is continuous.

Now assume that $f\colon \mathbb{N}_{\mathcal{I}} \to X$ is continuous. We want to show that $\mathcal{I}$-$\lim x_n = x$. Indeed, if U is an open neighborhood of x then $f^{-1}[U] = \{n : f(n) \in U\} \cup \{\infty\}$ is open in $\mathbb{N}_{\mathcal{I}}$, hence $\{n : x_n \in U\} \in \mathcal{F}(\mathcal{I})$ and $\{n : x_n \notin U\} \in \mathcal{I}$.

Let $\mathcal{S}$ be a family of proper ideals in $\mathbb{N}$. We say that a topological space X is *$\mathcal{S}$-sequential* if every map $f\colon X \to Y$ is continuous provided that f is $\mathcal{I}$-continuous for each $\mathcal{I} \in \mathcal{S}$. (We briefly say that f is *$\mathcal{S}$-continuous.*)

Lemma 3.2. *For each $\mathcal{I} \in \mathcal{S}$ the space $\mathbb{N}_{\mathcal{I}}$ is $\mathcal{S}$-sequential.*

Proof. Obviously $\mathcal{I}$-$\lim n = \infty$ in $\mathbb{N}_{\mathcal{I}}$. The assertion follows from Proposition 3.1.

Lemma 3.3. *The class of all $\mathcal{S}$-sequential spaces is closed under the formation of topological sums and quotient spaces. (I.e. $\mathcal{S}$-sequential spaces form a coreflective subcategory of* **Top**.)

Proof. Let X_j, $j \in J$, be a system of $\mathcal{S}$-sequential spaces, we want to show that $\coprod_{j\in J} X_j$ is $\mathcal{S}$-sequential. Let $f\colon \coprod_{j\in J} X_j \to Y$ be $\mathcal{S}$-continuous. It suffices to show that $f|_{X_j}$ is $\mathcal{S}$-continuous for each $j \in J$. (Then each $f|_{X_j}$ is continuous and therefore also f is continuous.) But it is clear that if $\mathcal{I}$-$\lim x_n = x$ in X_j then $\mathcal{I}$-$\lim x_n = x$ in $\coprod_{j\in J} X_j$ as well. So $\mathcal{I}$-$\lim f|_{X_j}(x_n) = \mathcal{I}$-$\lim f(x_n) = f(x) = f|_{X_j}(x)$.

Let X be a $\mathcal{S}$-sequential space and $q\colon X \to Y$ be a quotient map. Let $f\colon Y \to Z$ be $\mathcal{S}$-continuous. Then $f \circ q$ is $\mathcal{S}$-continuous (Proposition 2.1 and Theorem 2.2). Hence $f \circ q$ is continuous and f is continuous.

The next lemma is well-known in general topology.

Lemma 3.4. *Let X and Y be topological spaces and let $f\colon X \to Y$ be a surjective continuous map. Then f is quotient if and only if for each $g\colon Y \to Z$ it holds*

$$g \text{ is continuous} \quad \Leftrightarrow \quad g \circ f \text{ is continuous.} \tag{1}$$

Theorem 3.5. *A topological space X is $\mathcal{S}$-sequential if and only if it is the quotient of a topological sum of copies of spaces $\mathbb{N}_{\mathcal{I}}$, $\mathcal{I} \in \mathcal{S}$. (I.e. $\mathcal{S}$-sequential spaces are the coreflective hull of $\{\mathbb{N}_{\mathcal{I}}; \mathcal{I} \in \mathcal{S}\}$.)*

Proof. $\boxed{\Leftarrow}$ This implication follows from Lemma 3.2 and Lemma 3.3.

$\boxed{\Rightarrow}$ According to Proposition 3.1 a map $f\colon X \to Z$ is continuous if and only if for each $\mathcal{I} \in \mathcal{S}$ and every continuous map $g\colon \mathbb{N}_{\mathcal{I}} \to X$ the map $f \circ g$ is continuous. Let $g_j\colon X_j \to X$, $j \in J$, be the system of all continuous maps such that $X_j = \mathbb{N}_{\mathcal{I}}$ for some $\mathcal{I} \in \mathcal{S}$. Then the combination $q = [g_j]\colon \coprod_{j\in J} X_j \to X$ is quotient by Lemma 3.4.

By [2, Corollary 3.4] if $\mathbf{K}$ is a class of prime spaces and $X \in \mathrm{CH}(\mathbf{K})$ (i.e. X is a quotient space of topological sum of spaces belonging to $\mathbf{K}$), then every closed (open) subspace belongs to $\mathrm{CH}(\mathbf{K})$. So we get that every (closed) open subspace of a $\mathcal{S}$-sequential space is again $\mathcal{S}$-sequential.

A topological space is called *countable generated* if $V \subset X$ is closed whenever for each countable subspace U of X $V \cap U$ is closed in U. Countable generated spaces form the coreflective hull of countable spaces (see [6]). I.e. a space X is countable generated if and only if it is a quotient space of a topological sum of countable spaces.

It is known that each topological space X can be obtained as a quotient of a topological sum of (Hausdorff) prime spaces with cardinality not exceeding card X. X is a quotient of sum of so called prime factors of X, see e.g. [2]. Prime factors of X are prime spaces of the same cardinality as X. If a prime space P is not Hausdorff, then it is either finitely generated (and $\mathbf{FG} = \mathrm{CH}(S) \subset \mathrm{CH}(\mathbb{N}_{\mathcal{I}})$ for each admissible ideal $\mathcal{I}$ in $\mathbb{N}$) or it can be obtained as a quotient space of sum of a Hausdorff prime space P' (which is subspace of P) and several copies of the Sierpiński space S.

Corollary 3.6. *Let $\mathcal{S}$ be the system of all (admissible) ideals in $\mathbb{N}$. Then a topological space X is countable generated if and only if X is $\mathcal{S}$-sequential, i.e. for every topological space Y and every map $f\colon X \to Y$ the following holds:*

$$f \text{ is continuous} \Leftrightarrow f \text{ is } \mathcal{I}\text{-continuous for each (admissible) ideal } \mathcal{I} \text{ in } \mathbb{N}.$$

Example 3.7. Let A_j, $j \in J$, be a decomposition of $\mathbb{N}$ consisting of infinite sets. Let us define an ideal $\mathcal{I}$ in $\mathbb{N}$ such that M belongs to $\mathcal{I}$ if and only if M has infinite intersection with only finitely many A_j's. $\mathcal{I}$ is obviously an admissible ideal. (The space $\mathbb{N}_{\mathcal{I}}$ is homeomorphic to the space S_2^- from [5]. It is used in that paper as an example of a non-sequential space.)

We claim that no sequence of natural numbers converges to ∞ in $\mathbb{N}_{\mathcal{I}}$. Assume, indirectly, that $x_n \in \mathbb{N}$ for each $n \in \mathbb{N}$ and $\lim_{n\to\infty} x_n = \infty$ in $\mathbb{N}_{\mathcal{I}}$. For each $k \in \mathbb{N}$ the set $\mathbb{N}_{\mathcal{I}} \setminus A_k$ is an open neighborhood of ∞, so A_k contains only finitely many terms of the sequence $(x_n)_{n=1}^{\infty}$. Therefore $\{x_n : n \in \mathbb{N}\}$ belongs to $\mathcal{I}$ and the complement of this set is an open neighborhood of ∞ in $\mathbb{N}_{\mathcal{I}}$ which contains no terms of the sequence $(x_n)_{n=1}^{\infty}$.

So there are no non-trivial convergent sequences in the space $\mathbb{N}_{\mathcal{I}}$. Therefore every map $f\colon \mathbb{N}_{\mathcal{I}} \to X$ is $\mathcal{I}_f$-continuous. We construct a map $f\colon \mathbb{N}_{\mathcal{I}} \to \mathbb{R}$ so that $f(n) = \frac{1}{k}$ if $n \in A_k$ and $f(\infty) = 1$. Obviously, f is not continuous. So this example shows that the implication in Theorem 2.2 is not reversible.

From Lemma 3.2 it follows that f is not $\mathcal{I}$-continuous. ($\mathcal{I}$-$\lim n = \infty$ in $\mathbb{N}_{\mathcal{I}}$, but it does not hold $\mathcal{I}$-$\lim f(n) = f(\infty)$.) This example show also that the implication in Theorem 2.3 cannot be conversed.

Let $\mathcal{S}$ be a system of admissible ideals in $\mathbb{N}$. We want to find out whether the $\mathcal{S}$-sequential spaces can be characterized by a similar condition as the sequential spaces:

$$\begin{array}{c} V \text{ is closed in } X \text{ if for each } \mathcal{I}\text{-convergent sequence } (x_n)_{n=1}^{\infty} \text{ of} \\ \text{points of } V, \text{ where } \mathcal{I} \in \mathcal{S},\ V \text{ contains all } \mathcal{I}\text{-limits of } (x_n)_{n=1}^{\infty} \end{array} \tag{2}$$

If, for each $\mathcal{I}$-convergent sequence $(x_n)_{n=1}^{\infty}$ of points of V, V contains all $\mathcal{I}$-limits of $(x_n)_{n=1}^{\infty}$, we say briefly that V is closed with respect to $\mathcal{I}$-limits.

Lemma 3.8. *If a topological space X fulfils (2) then it is $\mathcal{S}$-sequential.*

Proof. Let $f\colon X \to Y$ be a $\mathcal{S}$-continuous map and $V \subset Y$ be a closed set. We want to show that $f^{-1}[V]$ is closed. It suffices to show that if $x_n \in f^{-1}[V]$ for each $n \in \mathbb{N}$, $\mathcal{I} \in \mathcal{S}$ and $\mathcal{I}$-$\lim x_n = x$ then $x \in f^{-1}[V]$. The $\mathcal{S}$-continuity of f implies that $\mathcal{I}$-$\lim f(x_n) = f(x)$ and $f(x) \in \overline{V} = V$. Hence $x \in f^{-1}[V]$.

Lemma 3.9. *The class of all spaces X which fulfil* (2) *is closed under the formation of topological sums and quotient spaces.*

Proof. A subset of $\coprod_{j\in J} X_j$ is closed if and only if for each $j \in J$ its intersection with X_j is closed in X_j. If V is closed with respect to $\mathcal{I}$-limits in $\coprod_{j\in J} X_j$ then $V \cap X_j$ is closed with respect to $\mathcal{I}$-limits in X_j for each $j \in J$. Therefore this class of spaces is closed under the formation of topological sums.

Let X fulfil (2) and let $q\colon X \to Y$ be a quotient map. Let V be a subset of Y which is closed with respect to $\mathcal{I}$-limits for each $\mathcal{I} \in \mathcal{S}$. We have to prove that $q^{-1}[V]$ is closed. It suffices to show that $q^{-1}[V]$ is closed with respect to $\mathcal{I}$-limits for each $\mathcal{I} \in \mathcal{S}$. Let $x_n \in q^{-1}[V]$ for each $n \in \mathbb{N}$ and let $\mathcal{I}\text{-}\lim x_n = x$, where $\mathcal{I} \in \mathcal{S}$. q is continuous, so it is by Theorem 2.2 also $\mathcal{I}$-continuous. Therefore $\mathcal{I}\text{-}\lim q(x_n) = q(x)$ and $q(x) \in V$, $x \in q^{-1}[V]$.

Combining Lemma 3.8, Lemma 3.9 and Theorem 3.5 we get:

Proposition 3.10. *Let $\mathcal{S}$ be a system of admissible ideals in $\mathbb{N}$. Let for each $\mathcal{I} \in \mathcal{S}$ the space $\mathbb{N}_{\mathcal{I}}$ fulfils* (2)*. Then a topological space X is $\mathcal{S}$-sequential if and only if $V \subset X$ is closed whenever for each $\mathcal{I}$-convergent sequence $(x_n)_{n=1}^{\infty}$ of points of V, where $\mathcal{I} \in \mathcal{S}$, V contains all $\mathcal{I}$-limits of $(x_n)_{n=1}^{\infty}$.*

The next example shows that in general (2) does not hold for a prime space $\mathbb{N}_{\mathcal{I}}$.

Example 3.11. Let A_j, $j \in J$, be a decomposition of $\mathbb{N}$ consisting of infinite sets. We define an ideal $\mathcal{I}$ in $\mathbb{N}$ such that M belongs to $\mathcal{I}$ if and only if the intersection $M \cap A_1$ is finite and M has infinite intersection with only finitely many A_j's. Clearly, the subspace of the space $\mathbb{N}_{\mathcal{I}}$ on the set $A_1 \cup \{\infty\}$ is homeomorphic to $\mathbb{N}_{\mathcal{I}_f}$ and the subspace on the set $\{\infty\} \cup (\bigcup_{j=2}^{\infty} A_j)$ is homeomorphic to the space S_2^- from Example 3.7. Let $V = \bigcup_{j=2}^{\infty} A_j$, $V \notin \mathcal{I}$ therefore V is not closed in $\mathbb{N}_{\mathcal{I}}$. But V is closed with respect to $\mathcal{I}$-limits.

Assume, on the contrary, that there exists a continuous map $f\colon \mathbb{N}_{\mathcal{I}} \to V \cup \{\infty\}$ such that it maps only ∞ to ∞. Then the restriction $f|_{A_1 \cup \{\infty\}}$ corresponds to a sequence in S_2^- which converges to ∞. But we showed in Example 3.7 that there is no such sequence in S_2^-.

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