Discrete mathematics I. - Problem set 0

Problem sessions in the week of September 23, 2013
Prove the following.

1. $\sum_{k=0}^{n} k^{2}=1^{2}+2^{2}+3^{2}+\cdots+n^{2}=\frac{n(n+1)(2 n+1)}{6}$.
2. $\sum_{k=0}^{n} \frac{1}{2^{k}}=2-2^{-n}$.
3. If $2 \mid n$ then $4 \mid n^{2}$.

Decide whether the following formulas are tautologies using the truth tables:
4. $p \Rightarrow(q \Rightarrow p)$
5. $(p \Rightarrow q) \Rightarrow p$
6. $(p \Rightarrow q) \Leftrightarrow \neg(p \wedge \neg q)$
7. $\neg(p \wedge q) \Leftrightarrow(\neg p \vee \neg q)$

What is the meaning of some of these tautologies in everyday life? Give an illustrative example.

