Discrete mathematics I. – Problem set 0

Problem sessions in the week of September 23, 2013

Prove the following.

- 1.  $\sum_{k=0}^{n} k^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$ .
- **2.**  $\sum_{k=0}^{n} \frac{1}{2^k} = 2 2^{-n}$ .
- **3.** If 2|n then  $4|n^2$ .

Decide whether the following formulas are tautologies using the truth tables: 4.  $p \Rightarrow (q \Rightarrow p)$ 

- 5.  $(p \Rightarrow q) \Rightarrow p$
- **6.**  $(p \Rightarrow q) \Leftrightarrow \neg (p \land \neg q)$

7. 
$$\neg (p \land q) \Leftrightarrow (\neg p \lor \neg q)$$

What is the meaning of some of these tautologies in everyday life? Give an illustrative example.