Discrete mathematics I. – Homework 1

Problem sessions in the week of September 30, 2013

Prove the following:

1. $1 + 3 + 5 + \cdots + (2n-1) = n^2$.

2. $1^3 + 2^3 + 3^3 + \cdots + n^3 = [\frac{n(n+1)}{2}]^2$.

Let us define a sequence of natural numbers $F_1, F_2, \ldots, F_n, \ldots$ as:

$$F_1 = 1, \quad F_2 = 1, \quad F_n = F_{n-2} + F_{n-1} \quad \text{for} \quad n \geq 3.$$

(We will get a sequence 1, 1, 2, 3, 5, 8, 13, 21, ...) This sequence is called *The Fibonacci sequence.*

3. Prove that every fourth element in the Fibonacci sequence is divisible by three, i.e. $3|F_{4n}$.

4. Prove that for any integer $n \in \mathbb{N}$ are the elements F_n and F_{n+1} coprime, i.e. $gcd(F_n, F_{n+1}) = 1$.

5. Prove the following formula for the Fibonacci number F_n:

$$F_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}.$$

Decide whether are the following formulas tautologies:

6. $p \Rightarrow [(\neg q \wedge q) \Rightarrow r]$.

7. $(p \Rightarrow q) \Leftrightarrow [(p \wedge q) \Leftrightarrow p]$.

8. Decide whether the following statement holds: ,,John can do logic if and only if it is not true, that is not true, that John can do logic".

9. Decide whether the following statement holds: ,,If an integer a is divisible by three then the fact that a is not divisible by three implies that a is divisible by five.".

Bonus problems

10. Define the logical connective *or* ($\vee$) using *conditional* ($\Rightarrow$) and *negation* ($\neg$).

11. Define the logical connective *and* ($\wedge$) using connective *or* ($\vee$) and *negation* ($\neg$).