Discrete mathematics I. – Homework 2

Problem sessions in the week of October 7, 2013

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1. There are n lines in general position in the plane, i.e. no two are parallel nor do any three of them intersect in one point. Into how many parts do they divide the plane?

2. Write formally the statement „n is the largest integer", using only symbols for variables, existencial or universal quantifier, relation symbol greater than/less than (i.e. $p > q$ or $p < q$), relation symbol equality (i.e. $p = q$) and logical operators.

3. Write formally the statement „x is an odd prime".
In this problem you can use symbols for variables, symbols for operations of addition and multiplication, universal or existencial quantifier, relation symbol greater than/less than, relation symbol equality, logical operators.

4. Prove that using only *biconditional* ($\Leftrightarrow$) and *negation* ($\neg$) it is not possible to define either connective *or* ($\wedge$) nor the connective *and* ($\vee$).

5. Find the disjunctive normal form for the formula $p \wedge [q \vee (\neg p \wedge r)]$.

6. Show that: $(\neg(a \Rightarrow b) \Rightarrow b) \models a \Rightarrow b$.

7. a) Show that $a \Rightarrow b$ is a tautological consequence of the formula $\neg(a \Rightarrow b) \Rightarrow \neg a$.
b) Show that: $\{(\neg(a \Rightarrow b) \Rightarrow c), (\neg(a \Rightarrow b) \Rightarrow \neg c)\} \models a \Rightarrow b$.

8. Let T be a satisfiable set of formulas. Show that $T \models A \Rightarrow B$ if and only if $T \cup \{A\} \models B$.

Bonus problems

9. Let there be n planes in general position in three dimensional space, i.e. no two of them are parallel, every triple intersect in exactly one point and no four of them intersect in one point. Into how many pieces do they subdivide the space? What would be the situation for $(k-1)$-dimensional hyperplanes in k-dimensional space $\mathbb{R}^k$?

10.* Prove that every (not necessarily convex) n-gon can be divided into $n-2$ non-overlapping triangles.