Disrete mathematics I. – Homework 3

Problem sessions in the week of October 14, 2013

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1. Let T be a satisfiable set of formulae. Show that $T \cup \{A \vee B\} \models C$ if and only if simultaneously $T \cup \{A\} \models C$ and $T \cup \{B\} \models C$.

2. Let T be a satisfiable set of formulae. Show that if the set of formulae $T \cup \{\neg A\}$ is unsatifiable then $T \models A$. (the other implication was done during the lecture)

3. Show that the formula $\neg a$ can be infered from the premisses $\{\neg(a \Rightarrow b), \neg(a \Rightarrow b) \Rightarrow \neg a\}$, i.e. find a formal proof according to the definition.

4. Let D be any axiom of the propositional calculus and E is any formula. Find a proof of the formula $E \Rightarrow D$.

5. Let T be a set of formuale and A, B, C are any formulae.Show that if $T \vdash A \Rightarrow B$ and $T \vdash A \Rightarrow (B \Rightarrow C)$ then $T \vdash A \Rightarrow C$. Hint: use axiom (A2).

6. Show that for the formula $(A \Rightarrow B) \Rightarrow (\neg A \Rightarrow \neg B)$ it is not possible to find any proof in the formal system, i.e. it is not a theorem of porpositional calculus. Hint: has to be any formula in the sequence of the proof tautology?

7. Show that the formula $a \Rightarrow b$ can be deduced from premise $\{\neg(a \Rightarrow b) \Rightarrow \neg a\}$. Hint: show $\{\neg(a \Rightarrow b) \Rightarrow \neg a\} \vdash a \Rightarrow (a \Rightarrow b)$, use axiom (A2) and problem 3.

Compare with problem 7a) from the last homework.