

Discrete mathematics I. – Homework 4

Problem sessions in the week of October 21, 2013

Recall the axioms of propositional calculus and three theorems proven during the lecture:

- (A1) $\vdash A \Rightarrow (B \Rightarrow A)$
- (A2) $\vdash (A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C))$
- (A3) $\vdash (\neg B \Rightarrow \neg A) \Rightarrow (A \Rightarrow B)$
- (T0) $\vdash A \Rightarrow A$
- (T1) $\vdash \neg A \Rightarrow (A \Rightarrow B)$
- (T2) $\vdash \neg\neg A \Rightarrow A$

Prove the following formulae. You can use axioms, inference rule Modus Ponens, as well as the Deduction Theorem, previously proven theorems, etc.:

1. (T2') $\vdash B \Rightarrow \neg\neg B$.
2. (T4) $\vdash A \Rightarrow ((A \Rightarrow B) \Rightarrow B)$.
3. (T3) $\vdash (A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)$. Hint: show $A \Rightarrow B, \neg\neg A \vdash \neg\neg B$ using (T2) and (T2').
4. $(\neg B \Rightarrow A) \vdash (\neg A \Rightarrow B)$.
5. (T7) $\vdash \neg A \Rightarrow (\neg B \Rightarrow \neg(A \vee B))$. Hint: use the definition: $A \vee B := \neg A \Rightarrow B$, show $\neg A \vdash (\neg A \Rightarrow B) \Rightarrow B$ and use (T3).
6. (T8) $\vdash A \Rightarrow (\neg B \Rightarrow \neg(A \Rightarrow B))$.

Bonus problems

7. $\vdash (\neg B \Rightarrow \neg A) \Rightarrow ((\neg B \Rightarrow A) \Rightarrow B)$.
8. $\vdash (A \Rightarrow B) \Rightarrow ((\neg A \Rightarrow B) \Rightarrow B)$.