Discrete matemathematics I. – Homework 6

Problem sessions for the week of November 4, 2013

We say that
A is a *subset* of B ($A \subseteq B$) if $(\forall x)(x \in A \Rightarrow x \in B)$.
A is an *empty set* if it does not contain any element. We denote it as $\emptyset$.
C is an *union* of A and B ($C = A \cup B$) if $(\forall z)(z \in C \Leftrightarrow (z \in A \vee z \in B))$.
D is an *intersection* of A and B ($C = A \cap B$) if $(\forall z)(z \in D \Leftrightarrow (z \in A \wedge z \in B))$.
E is a *Cartesian product* of A and B ($C = A \times B$) if $(\forall z)(z \in E \Leftrightarrow (\exists x \in A)(\exists y \in B)(z = [x, y]))$, where $[x, y]$ is so called *ordered pair* of elements x and y.
F is a *set difference* of A and B ($C = A - B$) if $(\forall z)(z \in F \Leftrightarrow (z \in A \wedge z \notin B))$.
$\mathcal{P}(A)$ is a *power set* of set A if $(\forall z)(z \in \mathcal{P}(A) \Leftrightarrow z \subseteq A)$.

1. Show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

2. Find elements of the set $A \times B$ if $A = \{a, b, c\}$ and $B = \{x, y\}$.

3. Decide whether $(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$. If so, prove it; if not, find a counterexample.

4. For the following pairs of sets decide whether $B \subset A$:
a) $A = \{\{a, b\}, \{c, d\}, c, d\}$, $B = \{\{a, b\}, c\}$
b) $A = \{\{a, b\}, \{a\}, b, \emptyset\}$, $B = \{\{a\}, b, \{\emptyset\}\}$
c) $A = \{x \in \mathbb{R} : x > 0\}$, $B = \{x \in \mathbb{Z} : x > 0\}$

5. Find the power set $\mathcal{P}(A)$ for $A = \{a, b, c\}$.

6. Show the set equality $A - (B - C) = (A - B) \cup (A \cap C)$.

7. For sets A, B show: $(A \cup B) - (A \cap B) = \emptyset \Leftrightarrow A = B$.