Discrete mathematics I. – Homework 7

Problem sessions in the week of november 11, 2013

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1. For which sets A, B does hold $A \cap (B - A) = \emptyset$?

2. What is $A \times \emptyset$?

3. Suppose that $A \subset B$. Show that $\mathcal{P}(A) \subset \mathcal{P}(B)$.

4. What is $\emptyset - (\emptyset - A)$?

5. Let $\mathcal{U} \neq \emptyset$. For every subset $A \subseteq \mathcal{U}$ denote the set $\mathcal{U} - A$ as A^{c}. Prove the following identities:
a) $\emptyset^{\mathrm{c}} = \mathcal{U}$.
b) $\mathcal{U}^{\mathrm{c}} = \emptyset$.
c) $(A^{\mathrm{c}})^{\mathrm{c}} = A$.
d) $A \cup A^{\mathrm{c}} = \mathcal{U}$.

6. For the relation of inclusion $\subset$ on the power set $\mathcal{P}(\{a, b, c\})$ draw a diagram representing this relation. ($A \subset B$ if $A \subseteq B$ and $A \neq B$)

We say that a relation R is:
reflexive if $(\forall x)\,[x, x] \in R$,
symmetric if $[x, y] \in R \Rightarrow [y, x] \in R$,
transitive if $([x, y] \in R \wedge [y, z] \in R) \Rightarrow [x, z] \in R$,
irreflexive if $(\forall x)\,[x, x] \notin R$,
asymmetric if $[x, y] \in R \Rightarrow [y, x] \notin R$,
antisymmetric if $([x, y] \in R \wedge [y, x] \in R) \Rightarrow x = y$,
dichotomous if $(\forall x, y)([x, y] \in R \vee [y, x] \in R)$.

7. Let $R = \{[a, b], [b, c], [d, e], [d, f]\}$ be a relation on a set $A = \{a, b, c, d, e, f, g\}$. Complete R in such a way that the resulting relation is an equivalence relation and the number of ordered pairs added is minimal.

For the following relations decide which properties they possess:

8. Relation R on $\mathbb{Z}$ defined as $xRy \Leftrightarrow 3|(x - y)$.

9. Relation R on $\mathbb{Z}$ defined as $xRy \Leftrightarrow x^2 = y^2$.

10. Relation R on $\mathbb{Q}$ defined as $xRy \Leftrightarrow x - y \notin \mathbb{Z}$.

11. Relation R on $\mathcal{P}(Y)$ defined as $ARB \Leftrightarrow a \in A \cap B$, where a is a fixed element of the set Y.

Bonus problem

12. Let $\sim$ be a relation on the set of complex numbers $\mathbb{C}$ given by: $z_1 \sim z_2$ if and only if $|z_1| = |z_2|$. Check that $\sim$ is an equivalence relation and describe corresponding equivalence classes $\tilde{z}$, i.e. sets of all $z' \in \mathbb{C}$, for which $z \sim z'$. (The norma of a complex number $z = a + b \cdot i$ is defined as $|z| = \sqrt{a^2 + b^2}$.)