Discrete mathematics I. – Homework 10

Problem sessions in the week of December 2, 2013

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1. Order the set of all rational numbers $\mathbb{Q}$ into an injective sequence. (I.e. every rational number will occur in the sequence exactly once).

2. Find a bijection between sets $(0,1\rangle \times \langle 0,1)$ and $\{[x,y] \in \mathbb{R}^2; 0 < x^2 + y^2 \leq 1\}$.

3. Find a bijection between the set of all rational numbers and the set of all nonzero rational numbers. Does there exist such a bijection which preserves ordering (i.e. if $x < y$ then $f(x) < f(y)$ as well)?

4. Let $A_1, A_2, \ldots$ be sets such that for every n is the intersection $A_1 \cap A_2 \cap \ldots \cap A_n$ nonempty. Could it happen that $A_1 \cap A_2 \cap \ldots = \emptyset$?

5. Show that the set of all open intervals with the endpoints in rational numbers is countable.

6. Show that the set of all open intervals with the endpoints in real numbers is uncountable.

7. Prove that every system of mutually disjoint open intervals in $\mathbb{R}$ has to be countable.