Midterm, Discrete mathematics, 28.10.2009

Axioms and theorems of propositional calculus which may be used:

(A1) $\vdash A \Rightarrow (B \Rightarrow A)$	(T1) $\vdash \neg A \Rightarrow (A \Rightarrow B)$
(A2) $\vdash (A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C))$	(T2) $\vdash \neg\neg A \Rightarrow A$
(A3) $\vdash (\neg B \Rightarrow \neg A) \Rightarrow (A \Rightarrow B)$	(T2') $\vdash B \Rightarrow \neg\neg B$
(T0) $\vdash A \Rightarrow A$	(T3) $\vdash (A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)$

1. Sequence $\{a_n\}$ is defined as $a_{n+1} = 3a_n + 1$ for $n \geq 1$ and $a_1 = 5$. Show that $a_n = \frac{11 \cdot 3^{n-1} - 1}{2}$.

2. Show that for elements in Fibonacci sequence $\{F_n\}$ (i.e. $F_1 = 1$, $F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$) for $n \geq 2$ holds:

$$F_n^2 = F_{n+1}F_{n-1} + (-1)^{n+1}.$$

3. Determine whether is formula $p \vee q$ a logical consequence of formula $((p \Rightarrow q) \Rightarrow r) \Rightarrow p$.

4. Show that $\neg(B \Rightarrow A) \vdash B$. You can use theorems mentioned above, axioms, rule Modus Ponens and the Deduction Theorem.

5. Show that the following formula with quantifiers is logically true or find a counterexample:

$$(\forall x)(\Phi(x) \Rightarrow \Psi(x)) \Rightarrow ((\exists x)\Phi(x) \Rightarrow (\forall x)\Psi(x)).$$