Midterm, Discrete Mathematics, 2.12.2009

We say that a relation R is:

reflexive if $(\forall x)[x,x] \in R$,
antireflexive if $(\forall x)[x,x] \notin R$,
symmetric if $[x,y] \in R \Rightarrow [y,x] \in R$,
antisymmetric if $([x,y] \in R \wedge [y,x] \in R) \Rightarrow x = y$,
dichotomous if $(\forall x,y)([x,y] \in R \vee [y,x] \in R)$,
transitive if $([x,y] \in R \wedge [y,z] \in R) \Rightarrow [x,z] \in R$.

1. Show that for all sets A, B, C we have:

a) $A \cup B = A - B \Leftrightarrow B = \emptyset$,

b) $A - (C \cap B) = (A - C) \cup B \Leftrightarrow ((B \subseteq A) \wedge (A \cap C = \emptyset))$.

2. How many relations on a four element set can be simultaneously antisymmetric and dichotomous?

3. On the set $\mathbb{R} \times \mathbb{R}$ we have a relation $\sim$ given by:

$$[x_1, y_1] \sim [x_2, y_2] \Leftrightarrow x_1 y_1 = x_2 y_2.$$

Show that the relation $\sim$ is an equivalence relation, determine what are the equivalence classes for elements $[1,1]$ and $[0,0]$, draw a picture describing the partition of $\mathbb{R} \times \mathbb{R}$.

4. Determine whether there exists a partial ordering of five element set which has 2 maximal and one greatest element. If it does exist, draw its Hasse diagram; if it does not, give an explanation.

5. Let $f : X \to Y$ be a map and $A, B \subseteq X$ are subsets of X. Show that $f(A) - f(B) \subseteq f(A - B)$. Find an example where equality does not hold. ($f(A)$ is an image of the set A, i.e. $f(A) = \{z \mid \exists a \in A, f(a) = z\}$).