2 Just, Weese: Discovering Modern Set Theory II – Set-Theoretic Tools for Every Mathematician

2.13 Filters and ideals in partial orders

2.13.1 The general concept of a filter

13.3(a): Show that if $\mathcal{F}$ is a nonprincipal ultrafilter on a set X with $|X| = \varkappa$, then $\mathcal{F}$ is not $\varkappa^+$-complete.

We have $\{x\} \notin \mathcal{F}$ for each $x \in X$. Thus $X \setminus \{x\} \in \mathcal{F}$ (since $\mathcal{F}$ is ultrafilter) and $\varkappa^+$-completness implies $\emptyset = \bigcap_{x \in X} X \setminus \{x\} \in \mathcal{F}$, a contradiction.

Nevedel som 6/Exercise 13.15

2.13.2 Ultraproducts

2.13.3 A first look at Boolean algebras

2.14 Trees

Contents