Meyer Jerison: The set of all generalized limits of bounded sequences

notes from [J]

Notation

$M = \ell_\infty$
M^* = dual with the weak* topology
B^* = unit ball of this topology
$\mathcal{H}(S)$ = closed convex hull

Introduction

generalized limit = linear functional φ on M such that

(i) $x \geq 0 \Rightarrow \varphi(x) \geq 0$

(ii) $\varphi(Tx) = \varphi(x)$

(iii) $\varphi(1, 1, 1, \ldots) = 1$

In the presence of (i), condition (iii) is equivalent to $\|\varphi\| = 1$.

$$B^* = \{\varphi | |\varphi(x)| \leq \|x\| \text{ for all } x \in M\}$$

is compact and convex, and we are able to apply the Krein-Milman theorem to its subsets.

We will denote by Ω' the set of extreme points of B^* that satisfy condition (1), or equivalently, the collection of extreme points of the subsets of B^* which is determined by conditions (1) and (3).

References

[J] Meyer Jerison. The set of all generalized limits of bounded sequences. *Canad. J. Math.*, 9:79–89, 1957.