1 Euler's summation formula

$$\sum_{a\le k\le b} f(k) = \int_a^b f(x)dx + \sum_{k=1}^{n} \frac{B_k}{k!} f^{(k-1)}(x)|_a^b + R_m,$$
$$\text{where } R_m = (-1)^{n+1} \int_a^b \frac{B_m(\{x\})}{m!} f^{(m)}(x)dx$$

integers $a \le b$, integer $n \ge 1$.[1]

$$B_0 = 1 \; B_{=} - \frac{1}{2} \; B_2 = \frac{1}{6} \; B_4 = -\frac{1}{30} \; B_6 = \frac{1}{42} \; B_8 = -\frac{1}{30} \; B_3 = B_5 = \ldots = 0$$

Bernoulli numbers are defined by an implicit recurrence relation

$$\sum_{j=0}^{m} \binom{m+1}{j} B_j = [m = 0] \text{ for all } m \ge 0$$

Bernoulli polynomial

$$B_m(x) = \sum_k \binom{m}{k} B_k x^{m-k}$$

Knuth: *Concrete Mathematics*

2 Eulerova sumačná formula

Veta 2.1. *Nech nezáporná reálna funkcia f má v intervale $\langle 0, +\infty)$ spojitú deriváciu. Potom pre každé $n = 1, 2, \ldots$ platí*

$$\sum_{k=0}^{n} f(k) = \int_0^n f(t)dt + \frac{f(0) + f(n)}{2} + \int_0^n (t - [t] - \frac{1}{2}) f'(t)dt.$$

Šalát: *Nekonečné rady,* s.147

[1] Ma tam byt $k < b$ ci $k < b$?