1 Aritmetické funkcie

Ak $f\colon \mathbb{N} \to \mathbb{C}$ je multiplikatívna, tak $g\colon \mathbb{N} \to \mathbb{C}$ definovaná ako $g(n) = \sum\limits_{m|n} f(m)$ je multiplikatívna. Pre $n = p_1^{\alpha_1} + \ldots + f(p_k^{\alpha_k}))$ platí $f(n) = (1 + f(p_1) + \ldots + f(p_1^{\alpha_1}))\ldots(1 + f(p_k) + \ldots + f(p_k^{\alpha_k}))$.

1.1 Möbiova funkcia

$\mu\colon \mathbb{N} \to \mathbb{C}$ je definovaná ako $\mu(n) = \begin{cases} 1, & \text{ak } n = 1 \\ (-1)^r, & \text{ak } n = p_1 \ldots p_r \\ 0, & \text{inak.} \end{cases}$

μ je multiplikatívna funkcia. Číslo n je bez štvorcov $\mu(n) = \pm 1$.

$$\sum_{m|n} \mu(m) = \begin{cases} 1, & \text{ak } n = 1, \\ 0, & \text{ak } n > 1. \end{cases}$$

$$\sum_{m=1}^{\infty} \frac{\mu(m)}{m^2} = \frac{6}{\pi^2}$$

Möbius inversion formula: Ak $g(n) = \sum\limits_{m|n} f(m)$ pre ľubovoľné n, tak

$$f(n) = \sum_{m|n} \mu(m) g\left(\frac{n}{m}\right) = \sum_{m|n} \mu\left(\frac{n}{m}\right) g(m).$$

Ak $f(n) = \sum\limits_{m|n} \mu\left(\frac{n}{m}\right) g(m)$, tak $g(n) = \sum\limits_{m|n} f(m)$.

$$f(n) = \sum_{d|n} \mu\left(\frac{d}{n}\right) g(d) \Leftrightarrow g(n) = \sum_{d|n} f(d)$$

$$G(x) = \sum_{n=1}^{[x]} F\left(\frac{x}{n}\right) \Leftrightarrow F(x) = \sum_{n=1}^{[x]} \mu(n) G\left(\frac{x}{n}\right)$$

$$g(x) = \sum_{n=1}^{\infty} f(nx) \Leftrightarrow f(x) = \sum_{n=1}^{\infty} \mu(n) g(nx)$$

Posledná rovnosť platí, len ak tie rady konvergujú.

1.2 Eulerova funkcia

$$\sum_{m|n} \varphi(m) = n$$

$$\varphi(n) = \sum_{m|n} \mu(m) \frac{n}{m} = n \sum_{m|n} \frac{\mu(m)}{m}$$

$$\varphi(n) = n \prod_{j=1}^{r} \left(1 - \frac{1}{p_j}\right)$$

$$\frac{6}{\pi^2} < \frac{\sigma(n)\varphi(n)}{n^2} < 1$$

$$\varphi(1) + \varphi(2) + \ldots + \varphi(n) = \frac{3n^2}{\pi^2} + O(n \log n)$$

1.3 Ďalšie funkcie

Pre každé $\delta > 0$ je $d(n) = O(n^{\delta})$.
$\limsup \frac{\log d(n) \log\log n}{\log n} = \log 2$
$\theta(x) = \sum_{p \leq x} \log p \Rightarrow \theta(x) > Ax$
$d(1) + d(2) + \ldots + d(n) = n \log n + (2\gamma - 1)n + O(\sqrt{n})$
$\sigma(n) = O(n^{1+\delta})$ pre $\delta > 0$
$\sigma(1) + \sigma(2) + \ldots + \sigma(n) = \frac{\pi^2}{12} n^2 + O(n \log n)$
$\sigma(n) = O(m \log n)$
Ak n je zložené, tak $\sigma(n) > n + \sqrt{n}$

2 Užitočné vzorce

$1 + \frac{1}{2} + \ldots + \frac{1}{n} - \log n = \gamma + O\left(\frac{1}{n}\right)$
$\pi(n) = \frac{n}{\log n} + \frac{n}{\log^2 n} + \frac{2! n^3}{\log^3 n} + \frac{3! n}{\log^4 n} + O\left(\frac{n}{\log^5 n}\right)$
n-té prvočíslo: $p(n) = n \log n + n \log\log n - n + \frac{n \log\log n}{\log n} + O\left(\frac{n}{\log n}\right)$
$n! = \sqrt{2\pi n} \left(\frac{n}{e}\right)^n \left(1 + \frac{1}{12n} + \frac{1}{288n^2} - \frac{139}{51840n^3} + O\left(\frac{1}{n^4}\right)\right)$

Literatúra

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