Bounded topological groups

[Tka2, Tka1]

$\aleph_0$-bounded topological groups

ω-narrow

homomorphic image [AT, Proposition 3.4.2], product [AT, Proposition 3.4.3], subgroup [AT, Proposition 3.4.4]

Every Lindelöf topological group is ω-narrow. [AT, Proposition 3.4.6]

Every separable topological group is ω-narrow. (In fact, countable cellularity suffices.) [AT, Proposition 3.4.7, Corollary 3.4.8]

If a topological group G contains dense subgroup H such that H is ω-narrow, then G is also ω-narrow. [AT, Theorem 3.4.9]

[AT, Corollary 3.4.19]: Let G be an ω-narrow group. Then for every neighbourhood U of the identity in G, there exists a continuous homomorphism π of G onto a second-countable topological group H such that $\pi^{-1}(V) \subseteq U$, for some open neighbourhood V of the identity in H.

A topological group G is topologically isomorphic to a subgroup of the topological product of some family of second-countable groups if and only if G is ω-narrow. [AT, Theorem 3.4.23]

Closure properties

- H=hereditary (subgroups)
- CH=closed subgroups
- D="closure" (If a dense subgroup has this property, so does the whole group.)
- FP=finite products
- P=arbitrary products
- HI=homomorphic image

	H	CH	D	FP	P	HI
ω-narrow	+	+	+	+	+	+
o-bounded	+	+	-	-	-	+
strictly o-bounded	+	+	-		-	+

ω-narrow H: [AT, Proposition 3.4.4] ($\Rightarrow$ CH)
D: [AT, 3.4.9]
P: [AT, Proposition 3.4.3] ($\Rightarrow$ FP)
HI: [AT, Proposition 3.4.2]

o-bounded H: [H, Theorem 2.1] ($\Rightarrow$ CH)
$\neg$P: $\mathbb{R}^\omega$ (with product topology) [H, Example 2.6]
$\neg$FP: [HRT, Example 2.12]
$\neg$D: σ-product in $\mathbb{R}^\omega$ (with product topology) [H, Example 2.6]
HI: [H, Theorem 2.3]

strictly o-bounded H: [H, Theorem 2.1] ($\Rightarrow$ CH)
$\neg$P: $\mathbb{R}^\omega$ (with product topology) [H, Example 2.6]
$\neg$FP: [HRT, Example 2.12]
$\neg$D: σ-product in $\mathbb{R}^\omega$ (with product topology) [H, Example 2.6]
HI: [H, Theorem 2.3]

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