A COURSE IN MATHEMATICAL LOGIC

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Synopsis

0. Informal Introduction to Formal Logic

- Logic is a normative science examining the form, structure and laws of correct (i.e., logical) thought and inference, as they become manifest in spoken or written language, abstracting from (i.e., taking no regard of) the content of particular thoughts and judgements.
- The rise of logic in antic Greece — social conditions: democracy, legal system, commerce, rhetoric
- Sophists, Socrates' dialogues, Plato, Aristotle (Organon)
- Evolution in Middle Ages and modern times (theology, Leibniz, Boole, de Morgan, Bolzano, Frege, Pierce; 20. century: Peano, Russell, Hilbert, Post, Tarski, Mal'tsev, Gödel, …)
- Mathematical logic (mathematical methods + analysis of language and deductive structure of mathematical theories)

1. Propositional Calculus
(Zeroth-Order Logic)

1.1. Syntax and semantics of Propositional Calculus

- Language of Propositional Calculus
- Propositions and propositional forms
- Interpretations (truth assignments) and truth tables
- Tautologies
- Theories in propositional calculus, logical consequence (validity in a theory)
- Axioms of propositional calculus, the deduction rule Modus Ponens
- The notion of proof, provability

1.2. The problem of soundness (correctness) and completeness in Propositional Calculus

- The Soundness Theorem and criticism of its proof
- Formulation of the Completeness Theorem
- Provability of some tautologies
- The Deduction Theorem, consistent theories, the Contradiction Theorem
- Post Theorem — provability of all tautologies

1.3. The proof of the Completeness Theorem

- The second formulation of the Completeness Theorem
- Complete theories (completeness = maximal consistency)
- Interpretability of complete theories
- The Compactness Theorem

2. First-Order Logic
(Predicate Calculus)

2.1. Syntax and semantics of First-Order Logic

- First-order languages and their structures
- Terms and formulas
- Free and bounded occurrences of variables
- Interpretation of terms and satisfaction of formulas
- First-order theories and their models
- Examples of first-order theories and models
- Logical consequence (validity in a theory)
- Logical axioms and deduction rules
- The notion of proof and provability

2.2. The problem of soundness (correctness) and completeness in First-Order Logic

- The Soundness Theorem and criticism of its proof
- Formulation of the Completeness Theorem
- The Deduction Theorem, consistent theories, the Contradiction Theorem
- Gödel's formulation of the Completeness Theorem
- Complete theories (completeness = maximal consistency)
- Henkin theories
- Existence of models of complete Henkin theories
- Skolem paradox
- The Compactness Theorem and its applications

3. Gödel's Incompleteness Theorems

- Natural numbers and Peano Arithmetics
- The problem of consistency and completeness
- Truth and provability
- Coding of sequences
- Arithmetization of provability
- The Epimenidus paradox (liar's paradox) and its shift
- First Gödel Incompleteness Theorem — incompleteness of PA
- Second Gödel Incompleteness Theorem — unprovability of consistency of PA
- Philosophical consequences

References

B. Balcar, P. Štěpánek, *Teorie množin.*

L. Bukovský, *Množiny a všeličo okolo nich.*

H. B. Enderton, *A Mathematical Introduction to Logic.*

T. Franzen, *Gödel's Theorem, An Incomplete Guide to Its Use and Abuse.*

J. Kolář, O. Štepánková, M. Chytil, *Logika, algebry a grafy.*

E. Mendelson, *Introduction to Mathematical Logic.*

P. Smith, *An Introduction to Gödel's Theorems.*

A. Sochor, *Klasická matematická logika.*

A. Sochor, *Logika pro všechny ochotné myslet.*

P. Štěpánek, *Matematická logika.*

V. Švejdar, *Logika – neúplnost, složitost, nutnost.*

P. Zlatoš, *Ani matematika si nemôže byť istá sama sebou. Úvahy o množinách, nekonečne, paradoxoch a Gödelových vetách.*